

# Mathematics

## Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

## Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

## Lower key stage 2 – years 3 and 4

The principal focus of mathematics teaching in lower key stage 2 is to ensure that pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.

Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.

# Year 3 programme of study

## *Number – number and place value*

### *Statutory requirements*

Pupils should be taught to:

- count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 1000
- identify, represent and estimate numbers using different representations
- read and write numbers up to 1000 in numerals and in words
- solve number problems and practical problems involving these ideas.

### *Notes and guidance (non-statutory)*

Pupils now use multiples of 2, 3, 4, 5, 8, 10, 50 and 100.

They use larger numbers to at least 1000, applying partitioning related to place value using varied and increasingly complex problems, building on work in year 2 (for example,  $146 = 100 + 40$  and  $6, 146 = 130 + 16$ ).

Using a variety of representations, including those related to measure, pupils continue to count in ones, tens and hundreds, so that they become fluent in the order and place value of numbers to 1000.

## *Number – addition and subtraction*

### *Statutory requirements*

Pupils should be taught to:

- add and subtract numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

### *Notes and guidance (non-statutory)*

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see [Mathematics Appendix 1](#)).

## *Number – multiplication and division*

### *Statutory requirements*

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects.

### *Notes and guidance (non-statutory)*

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example,  $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$ ) and multiplication and division facts (for example, using  $3 \times 2 = 6$ ,  $6 \div 3 = 2$  and  $2 = 6 \div 3$ ) to derive related facts (for example,  $30 \times 2 = 60$ ,  $60 \div 3 = 20$  and  $20 = 60 \div 3$ ).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which  $m$  objects are connected to  $n$  objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

## *Number – fractions*

### *Statutory requirements*

Pupils should be taught to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole [for example,  $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$ ]
- compare and order unit fractions, and fractions with the same denominators
- solve problems that involve all of the above.

### *Notes and guidance (non-statutory)*

Pupils connect tenths to place value, decimal measures and to division by 10.

They begin to understand unit and non-unit fractions as numbers on the number line, and deduce relations between them, such as size and equivalence. They should go beyond the  $[0, 1]$  interval, including relating this to measure.

Pupils understand the relation between unit fractions as operators (fractions of), and division by integers.

They continue to recognise fractions in the context of parts of a whole, numbers, measurements, a shape, and unit fractions as a division of a quantity.

Pupils practise adding and subtracting fractions with the same denominator through a variety of increasingly complex problems to improve fluency.

## **Measurement**

### *Statutory requirements*

Pupils should be taught to:

- measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)
- measure the perimeter of simple 2-D shapes
- add and subtract amounts of money to give change, using both £ and p in practical contexts
- tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks
- estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events [for example to calculate the time taken by particular events or tasks].

### *Notes and guidance (non-statutory)*

Pupils continue to measure using the appropriate tools and units, progressing to using a wider range of measures, including comparing and using mixed units (for example, 1 kg and 200g) and simple equivalents of mixed units (for example, 5m = 500cm).

The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or five times as high) and this connects to multiplication.

Pupils continue to become fluent in recognising the value of coins, by adding and subtracting amounts, including mixed units, and giving change using manageable amounts. They record £ and p separately. The decimal recording of money is introduced formally in year 4.

Pupils use both analogue and digital 12-hour clocks and record their times. In this way they become fluent in and prepared for using digital 24-hour clocks in year 4.

## *Geometry – properties of shapes*

### *Statutory requirements*

Pupils should be taught to:

- draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them
- recognise angles as a property of shape or a description of a turn
- identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle
- identify horizontal and vertical lines and pairs of perpendicular and parallel lines.

### *Notes and guidance (non-statutory)*

Pupils' knowledge of the properties of shapes is extended at this stage to symmetrical and non-symmetrical polygons and polyhedra. Pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate language, including lengths of lines and acute and obtuse for angles greater or lesser than a right angle.

Pupils connect decimals and rounding to drawing and measuring straight lines in centimetres, in a variety of contexts.

### *Statutory requirements*

Pupils should be taught to:

- interpret and present data using bar charts, pictograms and tables
- solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.

### *Notes and guidance (non-statutory)*

Pupils understand and use simple scales (for example, 2, 5, 10 units per cm) in pictograms and bar charts with increasing accuracy.

They continue to interpret data presented in many contexts.



# Year 4 programme of study

## *Number – number and place value*

### *Statutory requirements*

Pupils should be taught to

- count in multiples of 6, 7, 9, 25 and 1000
- find 1000 more or less than a given number
- count backwards through zero to include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare numbers beyond 1000
- identify, represent and estimate numbers using different representations
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.

### *Notes and guidance (non-statutory)*

Using a variety of representations, including measures, pupils become fluent in the order and place value of numbers beyond 1000, including counting in tens and hundreds, and maintaining fluency in other multiples through varied and frequent practice.

They begin to extend their knowledge of the number system to include the decimal numbers and fractions that they have met so far.

They connect estimation and rounding numbers to the use of measuring instruments.

Roman numerals should be put in their historical context so pupils understand that there have been different ways to write whole numbers and that the important concepts of zero and place value were introduced over a period of time.

## Number – addition and subtraction

### Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

### Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see [English Appendix 1](#)).

## Number – multiplication and division

### Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as  $n$  objects are connected to  $m$  objects.

### Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.

Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example  $600 \div 3 = 200$  can be derived from  $2 \times 3 = 6$ ).

### *Notes and guidance (non-statutory)*

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (see [Mathematics Appendix 1](#)).

Pupils write statements about the equality of expressions (for example, use the distributive law  $39 \times 7 = 30 \times 7 + 9 \times 7$  and associative law  $(2 \times 3) \times 4 = 2 \times (3 \times 4)$ ). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example,  $2 \times 6 \times 5 = 10 \times 6 = 60$ .

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

## **Number – fractions (including decimals)**

### *Statutory requirements*

Pupils should be taught to:

- recognise and show, using diagrams, families of common equivalent fractions
- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

### *Notes and guidance (non-statutory)*

Pupils should connect hundredths to tenths and place value and decimal measure.

They extend the use of the number line to connect fractions, numbers and measures.

Pupils understand the relation between non-unit fractions and multiplication and division of quantities, with particular emphasis on tenths and hundredths.

Pupils make connections between fractions of a length, of a shape and as a representation of one whole or set of quantities. Pupils use factors and multiples to recognise equivalent fractions and simplify where appropriate (for example,  $\frac{6}{9} = \frac{2}{3}$  or  $\frac{1}{4} = \frac{2}{8}$ ).

Pupils continue to practise adding and subtracting fractions with the same denominator, to become fluent through a variety of increasingly complex problems beyond one whole.

Pupils are taught throughout that decimals and fractions are different ways of expressing numbers and proportions.

Pupils' understanding of the number system and decimal place value is extended at this stage to tenths and then hundredths. This includes relating the decimal notation to division of whole number by 10 and later 100.

They practise counting using simple fractions and decimals, both forwards and backwards.

Pupils learn decimal notation and the language associated with it, including in the context of measurements. They make comparisons and order decimal amounts and quantities that are expressed to the same number of decimal places. They should be able to represent numbers with one or two decimal places in several ways, such as on number lines.

## **Measurement**

### *Statutory requirements*

Pupils should be taught to:

- Convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence

### *Statutory requirements*

- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

### *Notes and guidance (non-statutory)*

Pupils build on their understanding of place value and decimal notation to record metric measures, including money.

They use multiplication to convert from larger to smaller units.

Perimeter can be expressed algebraically as  $2(a + b)$  where  $a$  and  $b$  are the dimensions in the same unit.

They relate area to arrays and multiplication.

## *Geometry – properties of shapes*

### *Statutory requirements*

Pupils should be taught to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.

### *Notes and guidance (non-statutory)*

Pupils continue to classify shapes using geometrical properties, extending to classifying different triangles (for example, isosceles, equilateral, scalene) and quadrilaterals (for example, parallelogram, rhombus, trapezium).

Pupils compare and order angles in preparation for using a protractor and compare lengths and angles to decide if a polygon is regular or irregular.

Pupils draw symmetric patterns using a variety of media to become familiar with different orientations of lines of symmetry; and recognise line symmetry in a variety of diagrams, including where the line of symmetry does not dissect the original shape.

## Geometry – position and direction

### *Statutory requirements*

Pupils should be taught to:

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon.

### *Notes and guidance (non-statutory)*

Pupils draw a pair of axes in one quadrant, with equal scales and integer labels. They read, write and use pairs of coordinates, for example (2, 5), including using coordinate-plotting ICT tools.

## Statistics

### *Statutory requirements*

Pupils should be taught to:

- interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs.
- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.

### *Notes and guidance (non-statutory)*

Pupils understand and use a greater range of scales in their representations.

Pupils begin to relate the graphical representation of data to recording change over time.