Bishop Loveday C.E. Primary School

Calculation Policy

September, 2016

Introduction:
The fast recall of mathematical facts does not guarantee strong, capable, flexible mathematicians. Thinking interchangeably about number means that children can, when needed, apply a range of strategies when conducting any kind of calculation. *'Low achievers are often low achievers not because they know less, but because they don't use numbers flexibly – they have been set on the wrong path, often from an early age, of trying to memorise methods…'* (Boaler, 2015).

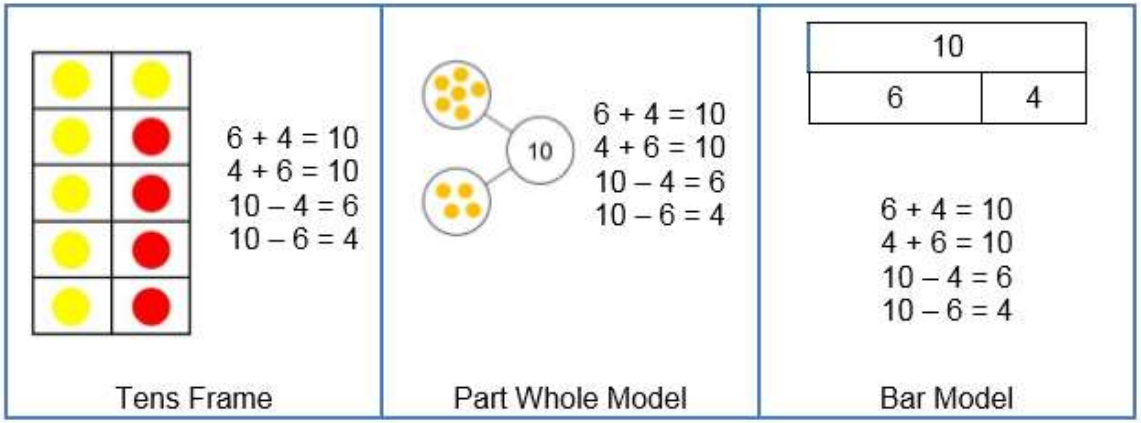
The National Curriculum for England (2014) makes clear that children:

- should become **fluent** in the fundamentals of mathematics – not the memorisation of facts, but the ability to work with numbers in a variety of ways, to complete calculations
e.g. Bridging through 10 (9 + 6 = 9 + 1 + 5 = 10 + 5 = 15) and
Place Value using Base Ten apparatus

- can **reason** mathematically - looking for patterns and connections to justify their thinking
e.g. What's the same and what's different about the 3 Times Table and the 6 Times Table?

-can **solve problems** by applying their mathematics to a variety of problems with increasing sophistication, including unfamiliar contexts and how the rules of arithmetic can be applied.

Maths is all about patterns and 'pattern sniffing' (finding the pattern). A sound way in which to ascertain children's understanding is to have them express their thinking in multiple ways using multiple resources and images (Grey & Tall, 1994; Ofsted, 2011), in ways that they determine to be useful.



Three different representations of 6 + 4

Given the National Curriculum requirement that, by the age of 9, children can recall their Times Tables, it is tempting for teachers to succumb to the routine drilling of children, through timed tests and repeated methods. A classroom, however, that is rich in resources that are: readily available; modelled by teachers; and self-selected by children will support children to believe that maths is accessible, relatable and, hopefully, interesting!

Children's fluency and conceptual understanding will be strengthened if they experience concrete, visual and abstract representations of a concept during a lesson. Moving between the concrete and the abstract helps children to ultimately make abstract connections with familiar objects.

Concrete Using actual resources	**Pictoral** Using images	**Abstract** Imagining and expressing
Cubes Counting sticks Counters* Cuisenaire rods Dienes (Base Ten apparatus) Place value counters Beads Numicon Ten frames (number bonds) Hundred squares Place value grids	Number lines (filled and blank) Bar models Pictures/ images Place value grids	

* Counters can be turned into place value counters by keep separating by colour and using a whiteboard pen to denote the value (i.e. TH, H, T, O)		

Why have a calculation policy?
By adopting a calculation policy, we ensure that children learn maths in ways that enable them to reason about concepts and make connections. Using the CPA approach, we ensure that children are not learning through mechanical repetition, but have opportunities to consistently practise their skills through multiple representations.

Acknowledgements
Many thanks to the White Rose Maths Hub for the use of its resources when developing this policy.

EYU to Year 1

There are fundamental skills that are important for children to develop, as these form the foundation for the later application of arithmetic.

A selection of the skills include:

- **Ordinality** – the ordering of numbers in relation to one another – e.g. (1, 2, 3, 4, 5…)
- **Cardinality** – the counting words are matched to the quantity of items
- **Equality** – '7 is the same total as 4 add 3' – e.g. 

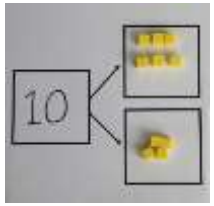
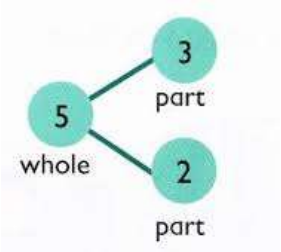
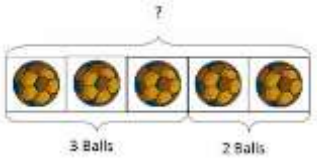
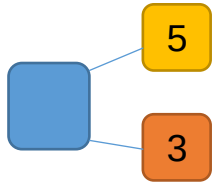
  Subitising – instantly recognising the number of objects in a small group, without counting them– e.g.  → 5
- **Conservation of number** – recognising that a value of objects are the same, even if they are laid out differently – e.g. 
- **One-to-one correspondence** – e.g. 

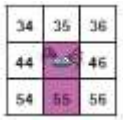
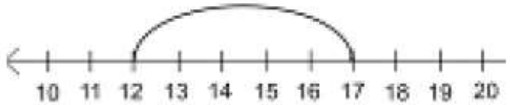
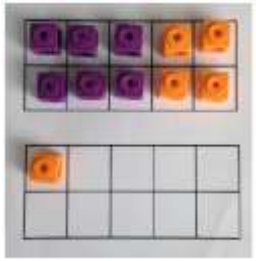
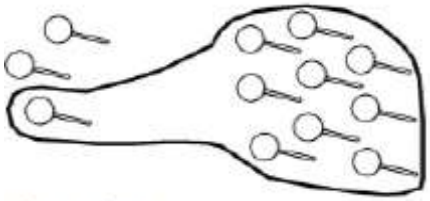
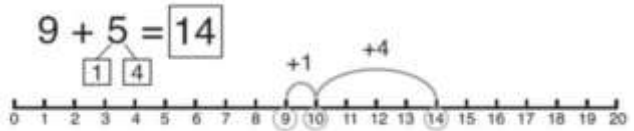
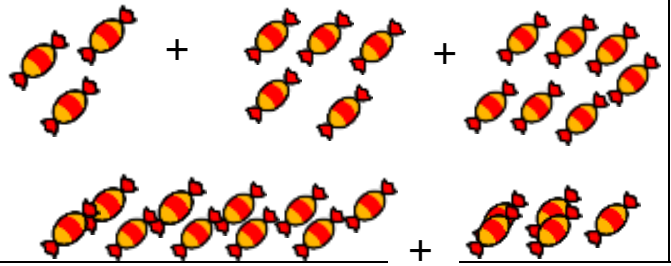
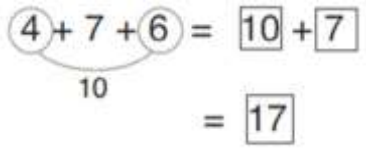
- **Counting on and back from any number** – e.g. 'Five add three more totals eight'  'Ten take away three is seven' 

- **Using apparatus and objects** to represent and communicate thinking – e.g. 

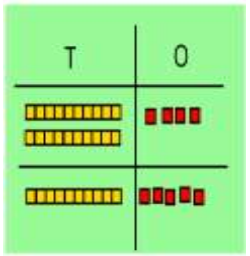
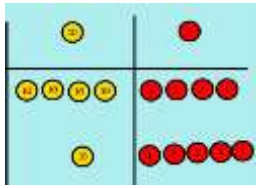
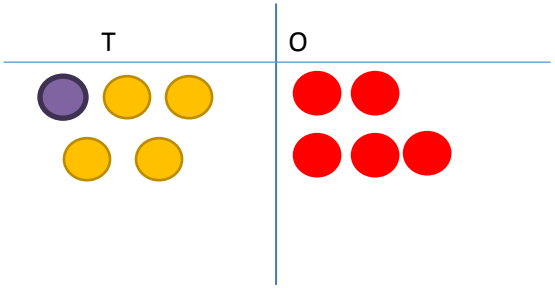
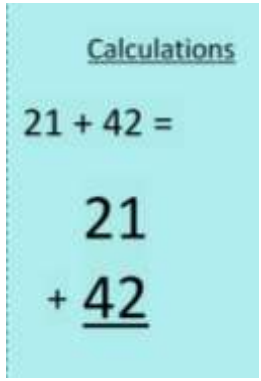
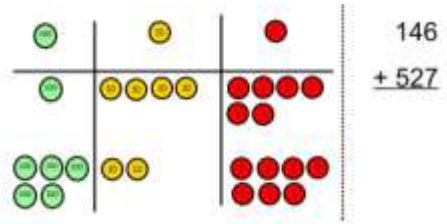
- **Maths language** – using mathematical words verbally in every-day situations – e.g. 'climb up to the top' / 'climb down to the bottom

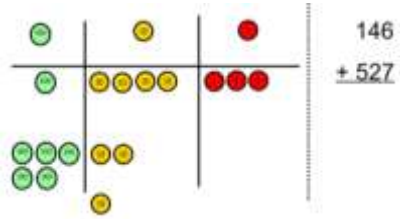
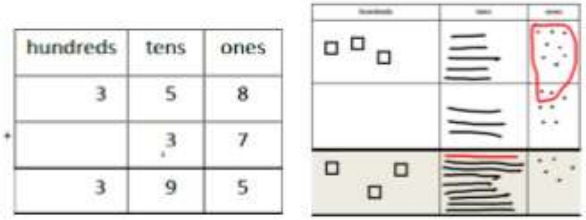
Addition:

Objective and Strategies	**Concrete** Using actual resources	**Pictoral** Using images	**Abstract** Imagining and expressing
Combining 2 parts to make a whole (part-part-whole model) <u>Useful resources</u> Multi-link Counters Straws Bar models 'Cherry' model Cuisenaire rods	   *Use cubes to add two numbers together as a group or in a bar.* 	   	4 + 3 = 7 10 = 6 + 4  *Use the part-part whole diagram, as shown above, to move into the abstract.*

Counting on from the larger number Useful resources Bead strings Multi-link Counters Straws Numicon Hundred squares Bar models Number lines	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  One more than four is five  + 10 without counting the ones	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping (to make 10) – The 'make 10' strategy Useful resources Bead strings Multi-link Ten frames Counters Straws Numicon Hundred squares (filled and blank) Images	 6 + 5 = 11  (Start with the bigger number and use the smaller number to make 10.	 Use pictures or a number line. Regroup or partition the smaller number to make 10. 3 + 9 = 	7 + 4= 11 If I am at 7, how many more do I need to make 10. How many more do I add on now?
Adding three single digits Useful resources Bead strings Multi-link Ten frames Counters	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.		 Combine the two numbers that

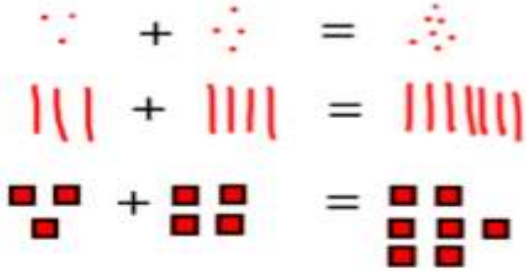
Add together three groups of objects.

Straws Numicon Hundred squares Images	 Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.		make 10 and then add on the remainder.
Column method with no regrouping (without creating groups of 10s) <u>Useful resources</u> Counters Base Ten Images Place value grids	24 + 15=  Use partitioning to add together the ones , before adding the tens. Use the Base 10 blocks first before moving onto counters. *Base Ten*  *Counters*	Draw counters to solve additions.  	
Column method- regrouping (using groups of 10s), to include work on decimals <u>Useful resources</u> Place value grids Counters Base Ten	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.	Children can draw a pictoral representation of the columns to further support their learning and understanding.	Start by partitioning the numbers before moving on to clearly show the exchange below the addition. 20 + 5 40 + 8 60 + 13 = 73 536 + 85 621 11

Money (decimals) Images	 146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base Ten to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.	 hundreds tens ones 3 5 8 3 7 3 9 5	Introduce decimals with the same number of decimal places (e.g to one decimal place). 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1 From this, you can start to look at the value of place holder. 2 3 . 3 6 1 9 . 0 8 0 5 9 . 7 7 0 + 1 . 3 0 0 9 3 . 5 1 1 2 1 2
Adding multiples of 10 <u>Useful resources</u> Base Ten Empty number lines Bar models Money Counters Counting sticks	50 = 30 + 20   3 tens + 5 tens = ____ tens 30 + 50 = ____ Use number sentences to support thinking.	Use vocabulary such as 1 Ten, 2 Tens etc, alongside 10, 20 etc.  	36 + 40 = ☐

Notes about

Support children to use known facts



3 + 4 = 7

leads to

30 + 40 = 70

leads to

300 + 400 = 700

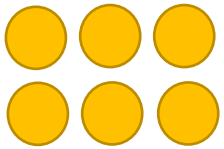
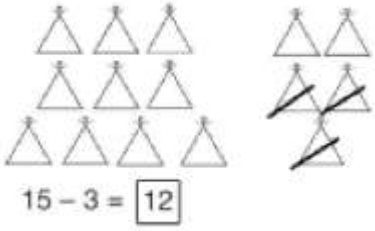
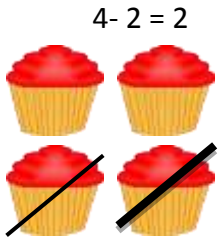
multiplication:

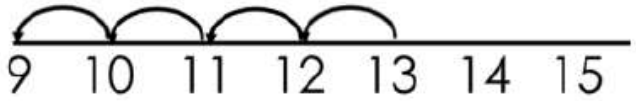
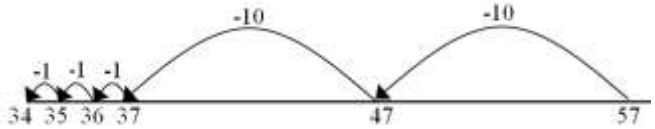
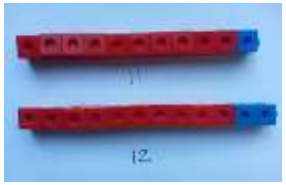
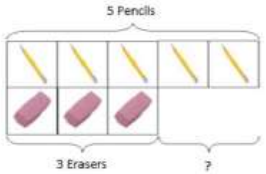
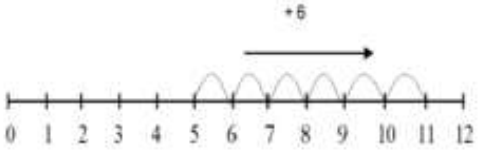
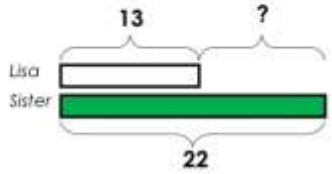
Times Tables are easier to learn in this sequence:

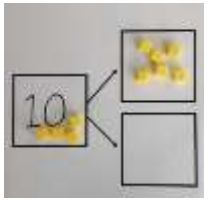
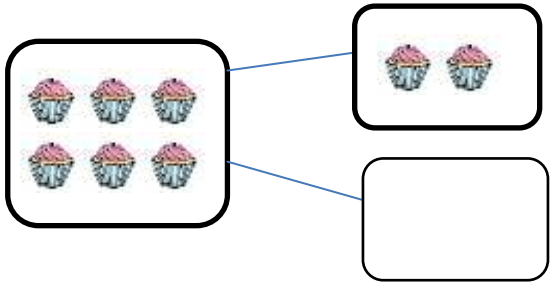
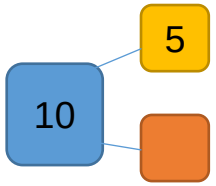
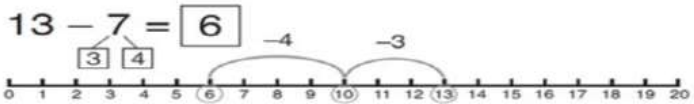
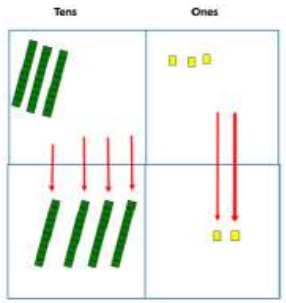
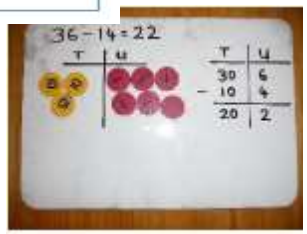
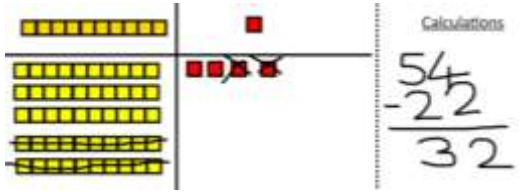
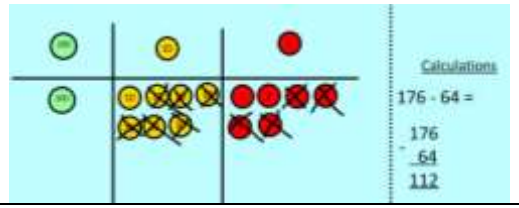
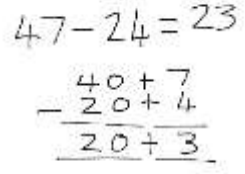
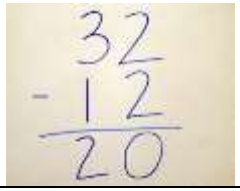
×10	×5	×2	×4	×8	×3	×6	×9	×7

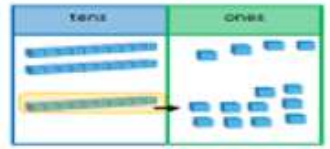
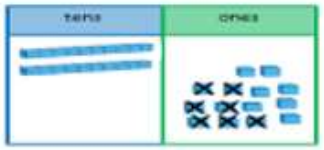
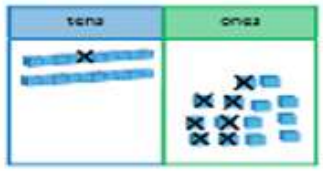
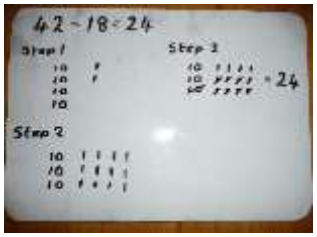
Due to the commutative nature of multiplication, once children know a set of facts, there are only 21 to learn (Times Tables, the 21 facts).

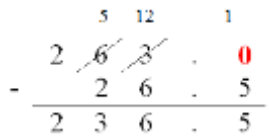
SUBTRACTION:

Objective and Strategies	Concrete	Pictoral	Abstract
Taking away ones <u>Useful resources</u> Bead strings Multi-link Counters Straws Counting sticks Number lines Images	Use physical objects, counters, cubes etc. to show how objects can be taken away.  6 – 2 = 4	Cross out drawn objects to show what has been taken away.  15 – 3 = 12  4- 2 = 2	18 -3 = 15 8 – 2 = 6

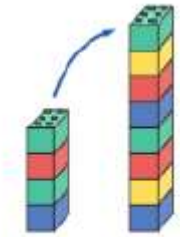
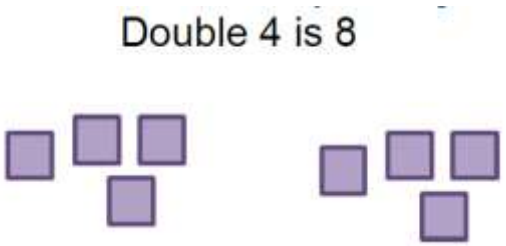
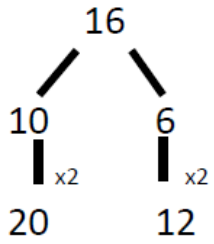
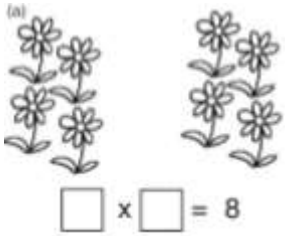
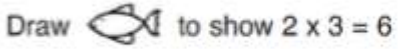
Counting back <u>Useful resources</u> Beads Counters Base Ten Counting stick Number lines (blank and filled)	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line. (Number lines also work well for lessons on Time)  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.
Finding the difference <u>Useful resources</u> Bead strings Multi-link Counters Straws Numicon Hundred squares Bar models	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference  	 Count on to find the difference. **Comparison Bar Models** *Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.*  Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15. Find the difference between the number of sandwiches.

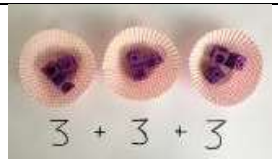
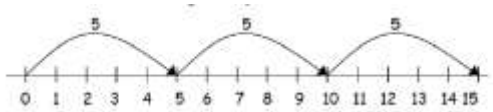
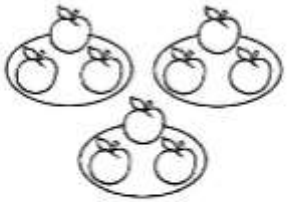
Part-part-whole model Useful resources Part-part whole images Counters Cherry tree Multi-link Images Bar model	 If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 = Link to addition - use the part-part-whole model to help explain the inverse between addition and subtraction.	Use a pictorial representation of objects to show the part-part whole model. 	 Move to using numbers within the part whole model.
'Make 10' strategy Useful resources Ten frames Multi-link Counters Number lines	14 – 9 =  Make 14 on the ten frame. Take away the 4first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Column method without regrouping (without creating groups of 10s) Useful resources Base Ten Counters Place value grids	 Use Base Ten to make the bigger number, then take the smaller number away. Show how you partition 	 Draw the Base Ten or counters alongside the written calculation to help show working. 	 This will lead to a clear  written column subtr

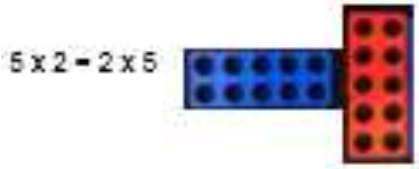
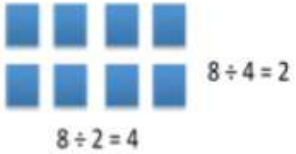
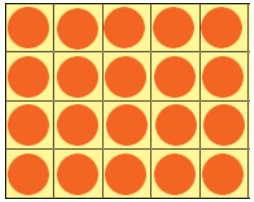
	numbers to subtract. Again, make the larger number first.		action.
Column method with regrouping (making groups of 10) <u>Useful resources</u> Place value grids Base Ten Counters	 34  34 – 17  34– (10 -7)  34 – 17 = 17 OR Make the number with counters, placing the larger number at the top, on a place value grid. Model the formal written method alongside the concrete method. When exchanging, show how 1 Ten is the same as ten Ones. Cross out the numbers, when	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident, children can find their own way to record the exchange/regrouping.  Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	 Children can start their formal written method by partitioning the number into clear place value columns.  Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.

	exchanging, and show where we write our new amount.		

Multiplication:

Doubling Useful resources Multi-link Counters Numicon Straws Images	Use practical activities to show how to double a number.  double 4 is 8 4 × 2 = 8	Draw pictures to show how to double a number.  	 Partition a number and then double each part before recombining it back together.
Counting in multiples Useful resources Multi-link Counters Numicon Straws Images Counting sticks Fingers Number lines Bar models	Count in multiples supported by concrete objects, in equal groups.  	  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

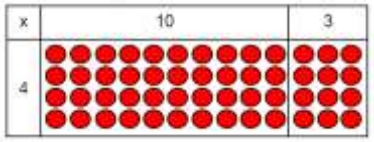
	Build tables on counting stick 		
Repeated addition Useful resources Multi-link Counters Numicon Straws Number lines Images Counting sticks	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15  How many apples are there altogether? 3 + 3 + 3 = 9	Write addition sentences, to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10

Commutative multiplication, using arrays Commutative = works either way (like a commuter) <u>Useful resources</u> Multi-link Counters Numicon Straws Images Cuisenaire rods	Create arrays using counters/ cubes to show multiplication sentences.    	Draw arrays in different rotations to find **commutative** multiplication sentences.  Link arrays to area of rectangles. 	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

Grid Method

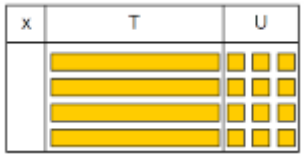
**The grid method is a progressive stage in formal multiplication. The National Curriculum (2014) does not recognise it as a written method, so it should be used to aid understanding before moving on to a formally recognised more efficient model.*

Show the link with arrays to first introduce the grid method.



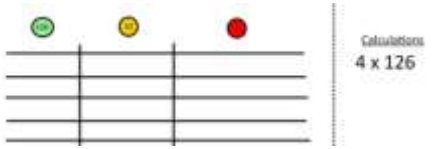
4 rows of 10
4 rows of 3

Move on to using Base Ten to move towards a more compact method.

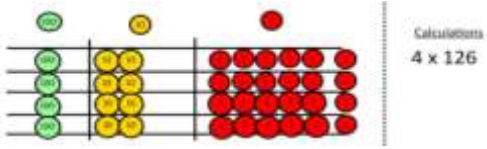


4 rows of 13

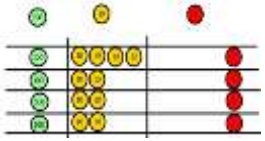
Move on to place value counters to show how we are finding groups of a number.We are multiplying by 4 so we need 4 rows.



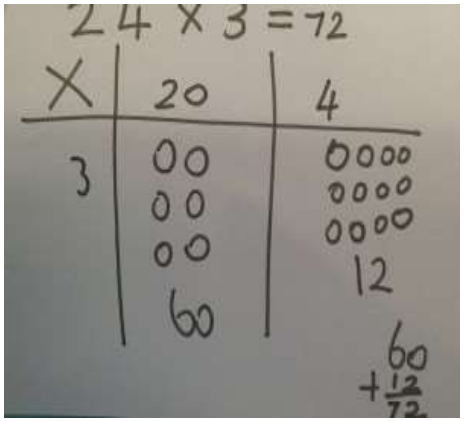
Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



Children can represent the work they have done with place value counters in a way that they understand.



They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

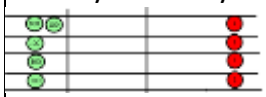
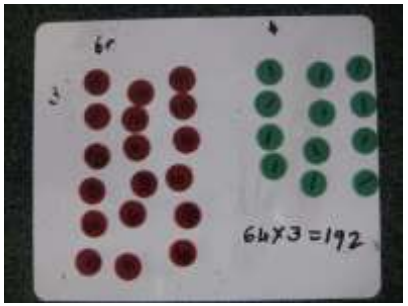
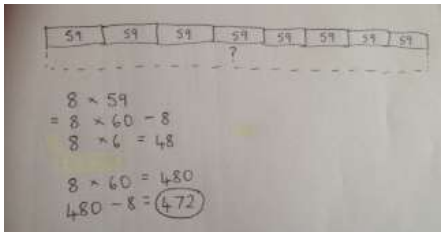
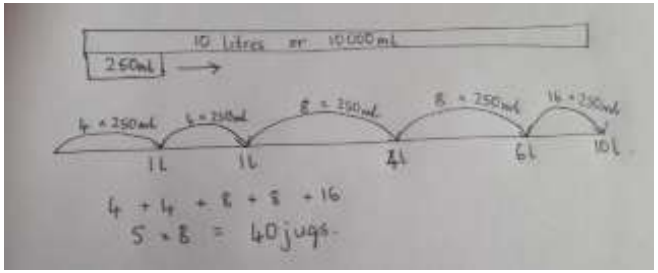
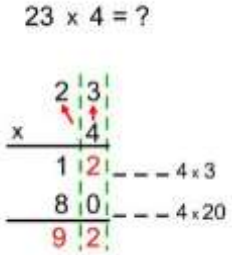
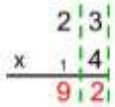
×	30	5
7	210	35

210 + 35 = 245

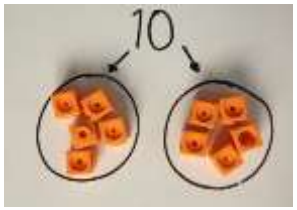
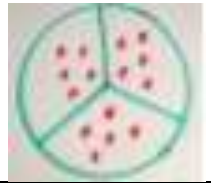
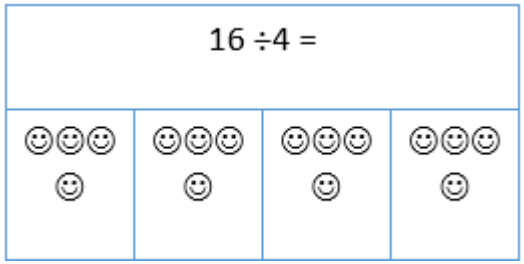
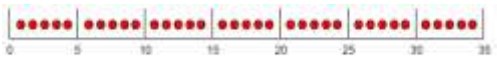
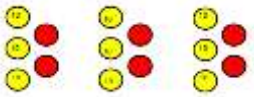
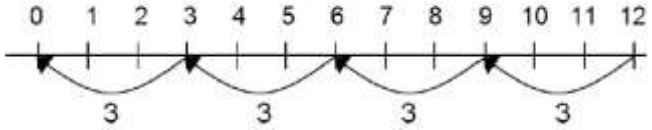
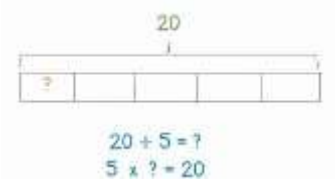
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

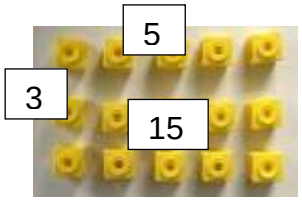
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

	Then you have your answer. 		
Column multiplication Useful resources Counters Bar models Empty number lines	Children can continue to be supported by counters at this stage of multiplication, should they need it.  It is important, at this stage, that children always multiply the 1s first and note down their answer, followed by the 10s, which they note below.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   	Start with long multiplication, reminding the children about lining up their numbers clearly in columns.  

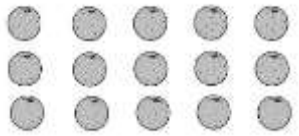
Division:

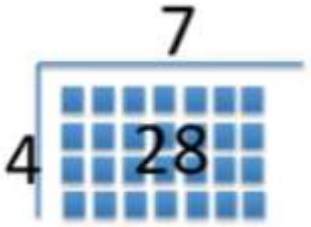
Division as sharing <u>Useful resources</u> Counters Cubes Objects Images Bar models	  I have 10 cubes. Can you share them equally, into 2 groups?  Link division to fractions 	Children use pictures or shapes to share quantities.  	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping <u>Useful resources</u> Cubes Counters Objects Beads Number lines Bar models	  96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

<table>
<tr>
<td>Division using arrays

<u>Useful resources</u>
Cubes
Objects
Images</td>
<td> 

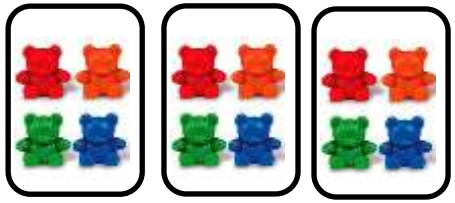
Link division to multiplication by creating an array and thinking about the number sentences that can be created.

e.g. 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15</td>
<td> 
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

 
</td>
<td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

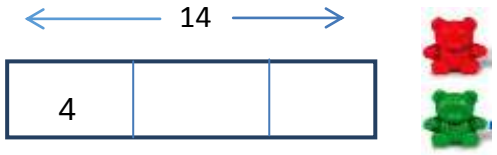
7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td>
</tr>
<tr>
<td>Division with a remainder

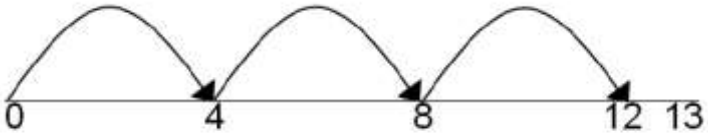
<u>Useful resources</u>
Cubes
Counters
Objects
Bar models
Number lines
Images</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over

OR

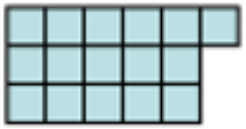
Give the answer (14 ÷ = 3 r2), showing children that you can remove the remainder and group using the bar model.

 
</td>
<td>Jump forward, in equal jumps on a number line, then see how many more you need to jump to find a remainder.

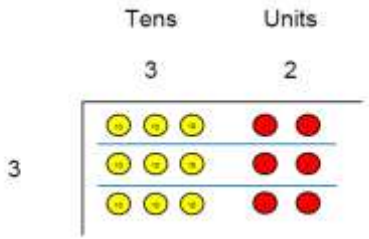
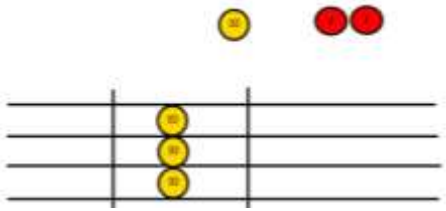
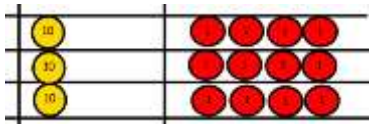
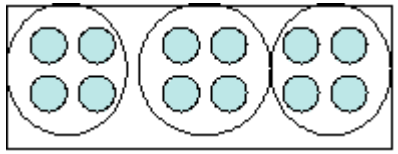
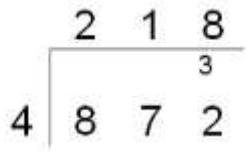
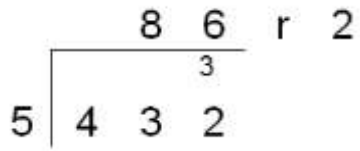
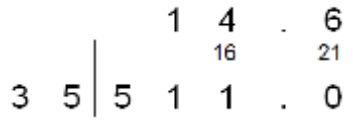
 

Draw dots and group them to divide an amount and clearly show a remainder.

 
16 ÷ 3 = 5 r 1</td>
<td>Complete written divisions and show the remainder using ‘r’.

28 ÷ 8 = 3 r5</td>
</tr>
</table>

Short division <u>Useful resources</u> Counters Base Ten Images Place value grids	Use counters to divide, using the bus stop method, alongside 42 ÷ 3 =  Start with the biggest place value, share 40 into three groups. Put 1 ten in each group, leaving 1 ten left over.  Exchange the left over ten for ten 1s and share these equally between three groups. Look at how much is in 1 group, so the answer is 14.  Base Ten blocks can be used in the same way.	Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally, with no remainder.  Move onto divisions, with a remainder.  Finally, move into decimal places to divide the total accurately. 

Important concepts for teaching (in alpahabetical order)	
Conjecture	Similar to a guess – something that requires proof
Decimal point	Decimal point is fixed, it's the digits that move
Ones	Rather than Units, as documented in the National Curriculum, 2014
Negative '6'	Use 'negative' rather than minus, to avoid confusion with other calculations
Prime number	A prime number has only two factors
Sum	Use solely for addition
Total	The sum found by adding